Maths and Facts: Introducing a Third Number

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Abstract

The "unreasonable effectiveness" of mathematics has appeared as something of a puzzle. I here suggest that the manner in which mathematics is applied to reasoning might profitably be disciplined by the introduction of the notion of P—an overt attribution of the locus of agency for the purposes of an ongoing discussion.

1   Introduction

The relationship of mathematics to the world we encounter and get by in has long been a source of wonder and puzzlement. Wigner famously extolled the "unreasonable effectiveness of mathematics" when applied within the natural sciences (Wigner, 1960). He quotes F. Werner, then a student, who asks

> "How do we know that, if we made a theory which focuses its attention on phenomena we disregard and disregards some of the phenomena now commanding our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory?"

. . . to which Wigner notes "It has to be admitted that we have no definite evidence that there is no such theory."

I wish to pursue here an entirely speculative course, bordering on the hallucinatory, that asks whether we might not be able to provide an addition to our discursive armoury that would allow us to discipline our use of mathematics in understanding the world, and thereby improve the quality of our collective discourse.

2   Mathematics, reduced

If we squint strongly, we can reduce the entire domain of mathematics to a few basic elements. There is the process of **deduction**, in which rules of some kind of logic are applied over primitives. When done correctly, novel statements can be made, and the nature of logical reasoning assures us that the certainty of the novel statements is not watered down by the process of deduction, so that the novel statements bear the same relationship to reality as do the premises from which they are deduced. The process of deduction does nothing, of course, to establish the utility, or truth, of the premises.

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The domain that supplies the primitives may be characterised in many ways. However because we need here to reduce things to a simplistic minimum, let us assert boldly that there are essentially two numbers that matter, and they are 1 and 0. We will consider each in turn, and then consider the introduction of a third term, worthy of being called distinct, that may serve us in using mathematics in talking about the world.

The number 1 can stand for the ability to individuate entities, thereby producing discrete elements that can be enumerated, and even ordered. All other counting numbers are nothing but extensions of the original move that distinguishes an entity as a discrete individual. A lot of mathematics can be done with this, now that we can count. Arithmetic, much of number theory, and many aspects of set theory are available.

A significant breakthrough happens with the addition of a zero. This is not another number of the first sort. It is an entirely new concept that serves to establish continuous metric spaces, thereby enlarging our set from basic arithmetic to now include the mathematics of continuous functions, topology, and so on. The whole of modern mathematics is now made possible.

It is perhaps worth noting that arguments about the relationship of mathematics to the world have long centred on whether the fundamental nature of reality is discrete or continuous. This was hotly disputed in ancient Greece. The existence of numbers that were not derivable from integers was a secret jealously guarded by the Pythagorians on pain of death. The Wave-Particle duality of modern physics assures us that such questions have not gone away and that they pertain to the nature of reality and how mathematics may be used to describe it. The difference between the concepts of individuated entities (the number 1) and continuous metric spaces (the reference point of 0) captures this ongoing duality.

3   Philosophical foundations

In order to establish a small philosophical platform to stand on, we will use the triumvirate of the two pre-Socratic philosophers, Heraclitus and Parmenides, and Socrates himself. As with our reduction of mathematics to a ridiculous minimalist sketch, so too we can can inflict a similar indignity on these three Gentlemen. Heraclitus reminds us that all we ever perceive, experience, or encounter is change. The present instant exists precisely because of the presence of change. Heraclitus valorises the now, and as a result, his concerns give rise to presentism with respect to time. We construct clock and calendar time as a framework to place such change in, and that requires the notion of state, which Parmenides helpfully provides. This gives rise to eternalism, in which the lawfulness that makes the world intelligible suggests that state evolves deterministically. Change, and the notion of now, have no place in Parmenides's cosmology.

Socrates provides a very useful third point from which we can encounter, and use, the insights of Heraclitus and Parmenides, but on the working assumption that we know nothing. We do not, and will not, take sides, but we can learn from both views. This is not unlike a position that recognises both aspects of wave-particle duality as necessary and complementary, but not resolvable within a single framework of description.

Descartes provides a necessary reference point for two, related, reasons. On the one hand, he accomplished the magnificent feat of abstraction that unified number theory and geometry, thus allowing discrete and continuous mathematics to co-exist. On the other, he viewed the observable properties of the world as utterly mechanical (before Newton) and his enduring legacy is a metaphysics that artificially cleaves the domains of the mental and the physical. Cartesian sub-

stance dualism is out of fashion, but as a methodological assumption, it still underpins the whole of psychology, psychiatry, and most received view of the person. The mathematical insight of Descartes, and the metaphysical legacy, are inextricably linked, and this squib hopes to provide a small means of going beyond this 350+ year old impasse. Descartes view of change is thus aligned with Parmenides, and he banished change that is of agentive origin to the wastelands of res cogitans.

Wittgenstein is taught to undergraduates as if he were two people. Early Wittgenstein was enamoured of logic and mathematics and its relation to the world. Later Wittgenstein provided a sophisticated non-referential account of language, thereby undercutting the simple relation many took to obtain between mathematics and the world—that of reference. What follows here seeks to unite early and late Wittgenstein by providing a non-referential account that nevertheless permits discussion of mathematics and the world in the same sentences.

4 Agency and the machine

Finally the meta-scientific observations of Bruno Latour will be relevant. Latour takes the position that the rigid distinction between subject and object, the Cartesian legacy, is poisoning our collective discourse. In order to deal with the many urgent issues facing mankind, it will be necessary to adopt a nuanced, negotiated, and polyvalent approach to existence, and to learn to recognise that many of the entities we passionately believe in and rely upon can be defended, must be defended, only by moving beyond a naive view of existence that recognises a domain of "physical" entities that are taken to have independent existence, and to relegate all the other institutions, beings, and practices to second-class existents (Latour, 2013). Latour too wants to move beyond Cartesianism, and argues that doing so is necessary if we are to rise to the challenges arising in the Anthropocene.

In a recent paper, he tellingly points the finger at our treatment of agency (Latour, 2014). Agency has no place in a mechanical view of the world. Apart from the motions of the inanimate, agents of one sort or another give rise to most of the change we see around us. Yet agency lies outside the established scientific framework. The success of the Newtonian framework for predicting motions of inanimate entities stands in contrast to its inability to say anything whatsoever about most of the motion we, animate, beings encounter in our animated biosphere. The acts of agents are, in a precise sense, meaningful. Existence as experienced by the living is intrinsically meaningful. It seems, as Latour laments, that the scientific world view (narrowly construed) has come to describe a "material world" in which the agency of all the entities making up the world has been made to vanish.

Recent developments within cognitive science have taken the view that the subjective unfolding of experience that we sometimes call consciousness has its origins, not in brains or information processing, but in the condition of life itself (Thompson, 2007; Deacon, 2012). On this view, the concerned perspective of the living gives rise to sense-making in which world and subject co-arise, a metaphysical perspective first introduced into cognitive science from Buddhist epistemology by Francisco Varela (Varela, 1993). This Mind and Life, or Enactive, approach grounds a theory of meaning in the purposeful activities of an agent. Agency thus lies at the very heart of meaning, and only agents can inhabit meaningful worlds. Heracletian in its outlook, and the manner in which the unfolding of experience in the present is addressed, the enactive approach likewise seeks to avoid insistence on any ontological split between subject and object. It is thus usually considered to be a constructivist philosophy.

5   A third number

It is here that there might be a space to move beyond 1 and 0, and to introduce a third term that is of help in tying mathematical reasoning to the world, and in doing so, to introduce a degree of self-consciousness in our attribution of agency. Let us call the new term P. In the context of any discussion, statements of fact must necessarily rest upon prior assumptions about the locus of agency. These assumptions career wildly as our everyday discourse ranges over the affairs of nations, football teams, natural forces, and the agents of good and bad luck. We do not normally bracket our discussions with explicit metaphysical considerations.

The role of P is to establish, for a given discussion, where agency is fixed *for the purposes of the present discussion.* Fixing a value of P thus allows us to establish what kind of statement can be granted the status of fact and what lies outside the frame of the discussion. The physics of Newton, for example, assumes a specific value of $P = 0$, that is, no statement made within the framework of Newtonian physics can lean upon any notion of agency whatsoever.

There is an implicit value of P when we create psychological models that treat human motion as mechanical. In the dynamical modeling of behavior, it has become clear that human movement often resembles the operation of a machine (Kelso, 1995). Blacksmiths hammering on an anvil, for example, resemble purpose-built hammering machines, in which one part of the system compensates rapidly for perturbations occurring at a distant location, as if all parts were designed to function with the goal of hammering in mind (Latash, 2008). It is possible to model many of the qualitative characteristics of behavior as if we were constantly reorganizing from one form of mechanical constitution to another. The domain of the mechanism changes all the time though: sometimes it may be evident that the fingers and palm of one hand are organized into a tapping machine; the whole body may function as a dancing machine in another context; the blacksmith-hammering machine incorporates the hammer and anvil as necessary constitutive components; two people engaged in amorous or aversive full body contact may together give rise to a two-person machine; many people may act in concert to give rise to a much larger mechanical phenomenon, e.g. in a Mexican wave (Farkas et al., 2002).

The adequate description of such quasi-machines depends upon identification of the boundary constraints that permit their establishment and maintenance. Some of these boundary constraints are straightforward and obvious: the rigidity of the anvil, for example, is necessary to permit hammering. Crucially, however, other constraints that serve to fix the phenomenon under observation are of an intentional, or goal directed, nature. We can understand these behaviours as mechanical only if we recognise that they come into being in the service of specific goals. The action of the hands in the service of a specific task demands that we recognise the task itself as part of the lens that allows us to see what we see; the task is thus an intentional object originating in the goals of an agent.

We might compare the selection of a specific value of P as akin to the selection of a specific temporal and spatial scale of observation within a conventional time-space framework. To see a cell as a dynamically individuated entity, we need to observe it at an appropriate scale. Too high a magnification, and we see only the inanimate processes of ion exchange, osmosis, etc. Too large a scale and we lose sight of the cell. If we select the wrong time scale, we will miss the cell too. But at the right time and spatial scale, we can recognise the cell as a persistent entity.

Likewise, when we select one value of P, attributing agency here, and not there, we make visible a set of facts that can stand in the context of the present discussion. These are not the lamentable notion of "facts that speak for themselves", as pilloried by Latour (Latour, 2013), but facts that

will serve in this context, and for the purposes of this discussion. In this manner, we can establish a link between the deductive processes of reason, and the premises over which reason ranges.

Fixing any value of P will make some phenomena visible, and will obscure others. This makes all discussions somewhat contingent, and tied to specific agreed assumptions. Positivists will not like this, but the approach will sit easily within constructivist discourse. In fact, the introduction of P is, more or less, the basic insight of second order cybernetics, which demands that the observer, or speaker, be aware of their own participation in observing or discussing. I like to think it addresses Socrates's concerns about the pernicious effect of the development of writing in which words and assertions are unleashed without the proper curation and care.

6   Where to go from here

The timeless eternalism of Parmenides has been a problem. To some, it has seemed like the inevitable consequence of an approach to natural law and lawfulness. The evolution of deterministic processes without qualification leaves no room for the agentive, meaningful lives we hope we are living. And it has no space for the unfolding of experience in the present. The constant churn of Heraclitus has offered little substantive to hold against the power of deterministic formulations. But like Socrates, we can stand back and grant both their insights. By introducing P to our basic toolbox that formerly held only 1 and 0, we can continue to do mathematics that describes lawful evolution of state, but we withhold our commitment to that state, refusing to grant it any existence outside the realm of our present discourse, as fixed and indexed by our attribution of agency, P. Early Wittgenstein's love of reason can be re-united with later Wittgenstein's awareness of the context-sensitive way in which meaning evolves, tied to our common understanding, our shared historicity, and our self-awareness of the manner in which we as discussants are entangled with the content of our discourse. Enlarging our box of mathematical foundations from two to three provides us with a way out of the inevitable split we inherited from Descartes in which number and geometry conspire to describe a world, but a world devoid of meaning and agency.

We live on Earth, essentially immersed in the webs of the living. This is our condition, and we must learn to live with it, as we also learn to shoulder responsibility for it. When we describe a Nature that exists independently of any observer, or a society that has an autonomous existence without qualification, we blind ourselves, and lose our own being in our description of things. Introducing P provides us with a manner of discipline in our assertions, a technology for framing our claims and assumptions, and a responsibility to conduct our discourse in a self-aware fashion, employing reason with care. This is not a mathematical insight, nor a biological, nor physical insight. It is political. As Latour says:

> Far from trying to reconcile or combine nature and society, the task, the crucial political task, is on the contrary to distribute agency as far and in as differentiated a way as possible until, that is, we have thoroughly lost any relation between those two concepts of object and subject that are of no interest any more except patrimonial. (Latour, 2014)

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